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MAGNUM MARANGONI FLUID FLOW ACROSS A CIRCULAR SURFACE: THE ROLE OF THERMAL RADIATION AND PRESSURE

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ABSTRACT

Taking into account the pressure work and surface tension, computational work has been carried out on a magneto Marangoni boundary layer that is thermally radiative on a heated circular surface. A system of ordinary differential equations describing various physical characteristics constitutes the numerical model of the present investigation, which was evolved from the governing partial differential equations with appropriate boundary conditions. Increasing the buoyancy parameter raises the magnitudes of the velocities, according to the numerical study of the effective variables, but the magnetic parameter and the ratio of the surface tension had the reverse impact. As the magnetic effect was amplified, the temperature magnitudes rose. The results showed that the Nusselt number is improved by the Marangoni ratio number, the Prandtl number, and the buoyancy parameter.

Thermoradiation, Marangoni fluid flow, pressure work, magnetic field, and circular surface are all relevant terms.

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1. Introduction

When two fluid surfaces have different surface tensions, a mass transfer process called the Marangoni consequence begins. Flow coat technology, microfluidics, foams, and film drainage in emulsions are just a few of the many industrial applications of this phenomena. Much study has gone into the Marangoni phenomena, and one finding from that study was that, for small Prandtl numbers, the Nusselt number (Nu) is proportional to (Pr)2/3 [1]. This finding was based on the Marangoni flow across a stretched surface. The non-linear temperature gradient becomes linear under these circumstances of low Prandtl number values [2]. Conversely, (Nu) is directly proportional to (Pr)1/6 for very high values of (Pr). Pop et al. [3] numerically investigated the Marangoni convection flow, while Marangoni convection heat transfer across a liquid-vapor surface [4] was analytically analyzed. Reduced velocity, increased temperature, and increased concentration are the results of an increase in thermo-solute surface tension [5]. A combination of a lower Soret number and an increase in the Dufour number causes the temperature to rise and the concentration to fall. Increasing the Prandtl number also improves the Nusselt number [6], and raising the Dufour number together with the nanofluid effect improves the heat transfer rates [7].

A lot of studies looked at the properties of thermal radiation for fluids flowing across diverse surfaces as it's relevant in a lot of engineering applications including nuclear power plants and space spacecraft. The nano-momentum boundary barrier becomes thinner as the combined influence of the magnetic field and the nanoparticle volume fraction grows, whereas the nano-thermal boundary layer becomes thicker. The local Nusselt number also goes down when the magnetic parameter becomes up [8]. In contrast to delaying the Darcy-Benard-Magneto Marangoni convection [10], the non-Darcy-Benard Marangoni convection [9] is caused by the presence of porosity and the Chandrasekhar parameter.

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To improve the Marangoni radiative effect on permeable surfaces with inclined magnetic fields, several nanoparticles provide an effective tool [11]. The velocity of nanofluids based on H2O and C2H6O2 both lowers when the radiation and heat source impacts are increased [12]. The skin friction coefficient drops when mixed convection increases, although temperature rises somewhat. On the other side, skin friction coefficients tend to be higher when porous media are present [13]. Devices operating in environments with strong gravitational fields or with accelerated rotational velocities are particularly vulnerable to the effects of pressure work. Thus, the impact of pressure stress on forced/free convection flow over radial surfaces, truncated cones, vertical flat plates, horizontal cylinders, and stretched surfaces has been thoroughly investigated. The pressure work was discovered to reduce the temperature and velocity profiles [14]. A larger skin friction coefficient is achieved by raising the nonlinearity, surface permeability, chemical reaction parameter, and Brownian motion parameters. Thermal radiation, chemical reactions, the thermophoresis effect, and the rate of mass transfer all rise as the amount of nanoparticles increases [15]. When applied to a shallow cylinder, Marangoni convection changes the flow [16]. Fluid velocity is reduced due to permeability effects when surface tension rises [17]. Here, the buoyancy parameter increases the flow rate and the porosity value regulates the magnitudes of the heat transmission characteristics [18]. In addition, the hybrid nanofluid is significantly affected by Marangoni convection [19]. Theoretically, taking into account the pressure work, prior research suggests a promising avenue for merging thermal radiative heat transfer with magneto Marangoni convection in a fluid flowing over a circular plate that is heated vertically. Surface tension and surface heating are examined for their impacts. Surface tension is useful in many contexts, including those that need steady flow across films and those encountered during integrated circuit fabrication.

2. Mathematical Model

A two-dimensional, laminar, stable and incompressible Marangoni boundary layer is currently examined, where the flow across a heated circular plate is under pressure and stress.

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The thermal gradient-driven Marangoni convective and radiative flow above a planar interface is considered. The r-axis is considered in the direction of motion which is perpendicular to the z-axis (Fig. 1). While the plane ($z = 0$) represents the surface, the flow is confined in the region wherein $z > 0$. A uniform magnetic field of strength B_0 is applied in the z-direction (which is perpendicular to the flow). The Marangoni phenomenon occurs when the mass transmission develops due to the variation of the surface tension at the interface of two surfaces. Such variation in the surface tension (which may result from a gradient in the temperature or in the concentration) causes the fluid to get transported from the lower interfacial tension to the higher one.

With the above-mentioned assumptions, the basic equations of flow are given as follows:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial z^2}\right)+g\beta_T(T-T_\infty)-\frac{\sigma^* B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial z^2}\right)+\frac{T\beta u}{\rho c_p}\frac{\partial P}{\partial r}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial z} \tag{3}$$

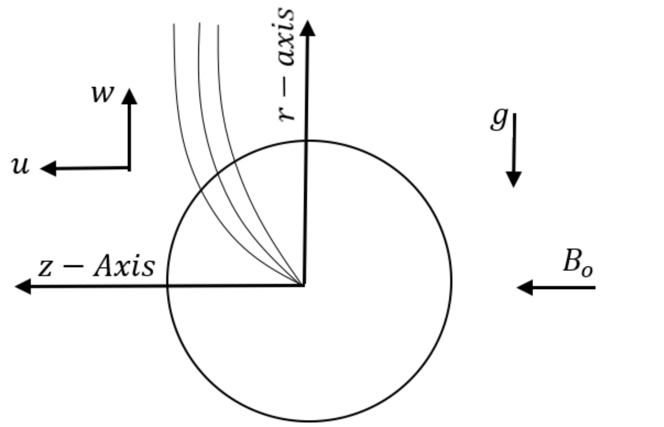

Fig. 1: Diagram illustrating the issue

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The boundary conditions are:

$$at\ z = 0:\ \ \mu\frac{\partial u}{\partial z} = \frac{\partial \sigma}{\partial r},\ \ \ w = 0,\ \ T = T_w,\ \ z \to \infty:\ u = 0, w = 0, T = T_\infty \qquad (4)$$

Where,

$$r\frac{\partial P}{\partial r} = \rho g \qquad (5)$$

By using Rosseland diffusion approximation

$$q_r = -\frac{4\sigma_1}{3k^*}\frac{\partial T^4}{\partial z} \qquad (6)$$

Taylor expansion is used to expand T^4 about T_∞ while ignoring the higher-order terms yields:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \qquad (7)$$

Therefore, equation (3) becomes

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = [\alpha + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*}]\left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{T\beta\rho g}{r\rho c_p}\,u \qquad (8)$$

Using Boussinesq approximation, the surface tension is considered as follows:

$$\sigma = \sigma_0[1 - \gamma(T - T_\infty)] \qquad (9)$$

Where σ_0 is the interface surface tension ratio and $\gamma = -\frac{1}{\sigma_0}\frac{\partial \sigma}{\partial T}$,

The stream function $\Psi\,(r, z) = -\frac{ur^2}{L} f(\eta)$ is chosen such that

$$u = -\frac{1}{r}\frac{\partial \psi}{\partial z}\ ,\quad w = \frac{1}{r}\frac{\partial \psi}{\partial r} \qquad (10)$$

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are satisfying the continuity equation. For similarity solution, the following variables are introduced:

$$\eta = \frac{z}{L} \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad T_w - T_\infty = T_0 r^2 \tag{11}$$

Where T_0 is the reference temperature constant, while the controlling equations can be written as follows:

$$f''' - f'^2 + 2ff'' - Mf' + \xi\theta = 0 \tag{12}$$

$$(1 + N)\theta'' + 2Prf\theta' - 2Prf'\theta - Pr\epsilon\,(\theta + \frac{1}{\theta_w - 1})\,f' = 0 \tag{13}$$

With the following boundary conditions:

$$z = 0: f'' = -2S, \quad \theta = 1 \; z \to \infty: f = 0, \quad \theta = 0 \tag{14}$$

Where

$$\zeta = \frac{g\beta(T_w - T_\infty)L^4}{rv^2}, M = \frac{L^2\sigma^* B_0^2}{\rho_\infty}, Pr = v/\alpha, \theta_w = \frac{T_w}{T_\infty}, N = \frac{16\sigma^* T_\infty^3}{3kk^*}, \epsilon = \frac{\beta vg}{c_p}, S = \frac{\sigma_0\gamma T_0}{\mu v} \tag{15}$$

The practical physical parameters are defined as follows:

$$Nu = \frac{rq_w}{k_f(T_w - T_\infty)} \tag{16}$$

Where q_W represents the surface heat flux which is thus given by

$$q_w = -k_f\left(\frac{\partial T}{\partial z} + q_r\right)_{z=0} = -\frac{(T_w - T_\infty)}{L} k_f(1 + N)\,\theta'|_{\eta=0} \tag{17}$$

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Finally, the following equation is used:

$$\frac{L}{r} Nu = -(1+N)\theta'(0) \tag{18}$$

3. Solution Methodology

Equations (12) and (13) are transformed into a system of linear equations. In this case, $y_1 = f, y_2 = f', y_3 = f'', \ y_4 = \theta, y_4 = \theta'$

$$y_1' = y_2 \tag{19}$$

$$y_2' = y_3 \tag{20}$$

$$y_3' = y_2^2 - 2y_1y_3 + \lambda y_2 - \xi y_4 \tag{21}$$

$$y_4' = y_5 \tag{22}$$

$$y_5' = \frac{1}{1+N}(2\,Pr\,(y_1y_4 - y_2y_4) + Pr\epsilon(y_4 + \frac{1}{\theta_w - 1})y_2) \tag{23}$$

Where the boundary conditions are given as follows:

$$y_1(0) = 0,\ y_2(0) = s_1,\ y_3(0) = -2S, \qquad y_4(0) = 1, y_5(0) = s_2 \tag{24}$$

Where s_1 and s_2 are arbitrary parameters. The equations from (19) to (23) are solved numerically by using 4th/5th Runge-Kutta method and Mathematica software.

4. Results and Discussion

A satisfactory agreement is obtained upon comparing the present work results with the results of Zhang and Zheng [20] as shown in Table 1, which correspond to the values of $f'(0)$. Such agreement testifies to the accuracy of the current work analysis.

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Table 1: Comparison values $f'(0)$ for various values of M at $\xi = 0$		
M	**Zhang and Zheng [20]**	**Present Work**
0.0	2.5199	2.5200
1.0	2.1572	2.1570
2.0	1.6240	1.6399

The effects of the M parameter on the fluid velocity boundary layers tends to produce a Lorentz force which causes a flow retardation effect. This causes the fluid velocity to decrease (as shown in Fig. 2a). On the other hand, enhancing the values of M enlarges the width of the thermal boundary layer, Fig. 2b. The influence of the buoyancy parameter is shown in Fig. 3. It is noted that the increase in the buoyancy parameter tends to increase the velocity. Physically, the buoyancy force is related to the inertial force. Figure 4 shows the temperature distribution for different values of the pressure work parameter (ϵ). The increase in the pressure work parameter causes an increase in the temperature. Figure 5 shows that as the value of Pr increases, the value of $\boldsymbol{\Theta}$ decreases. Physically, increasing Pr decreases the thermal diffusion such that the thickness of the thermal boundary layer decreases and the heat transfer rate becomes smaller.

Figure 6 shows that the temperature magnitudes increase as the N parameter increases. Therefore, the radiation heat transfer can be used to control the thermal boundary layers quite effectively where the thermal boundary layer thickness increases with the increase in the N parameter. Figure 7 indicates that the increase in the θ_w parameter increases the temperature. In Fig. 8, as the S parameter increases, more induced flow is produced within the boundary layer. It is noted that the temperature is reduced by involving a higher Marangoni convection parameter (S) but the opposite behavior occurs for the velocity profile. Physically, the Marangoni convection is inversely proportional to the viscosity. It means that the larger value of the Marangoni parameter is correlated to low viscosity which decelerates the fluid velocity. Table 2 shows that the Marangoni number S, Prandtl number and buoyancy parameter enhance the heat transfer rate, but the opposite result is obtained for the magnetic parameter, the Pressure work parameter, the radiation parameter and the surface heating parameter.

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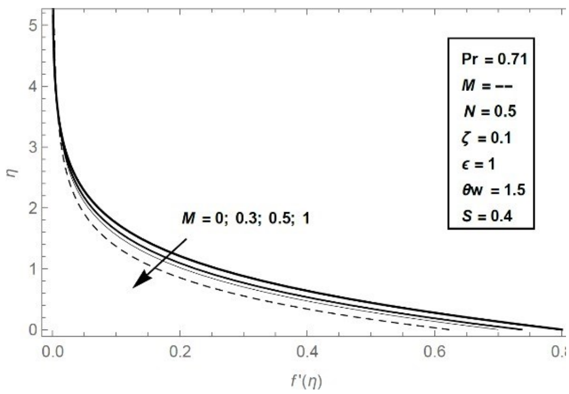

Fig. 2a. Velocity graph f' for M

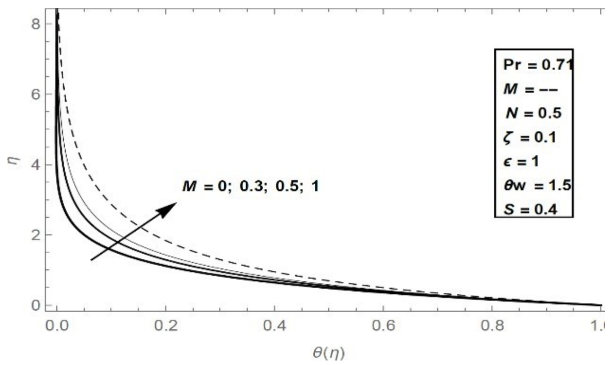

Fig. 2b. Temperature graph θ for M

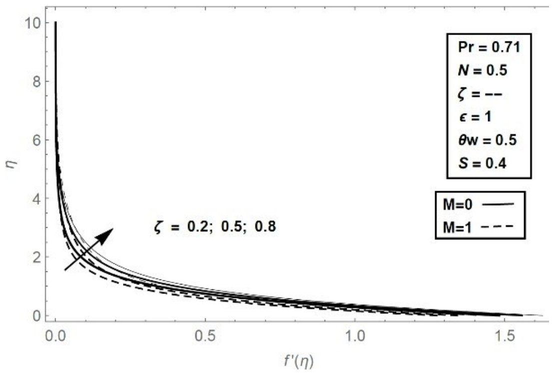

Fig. 3: Velocity graph f' for ζ

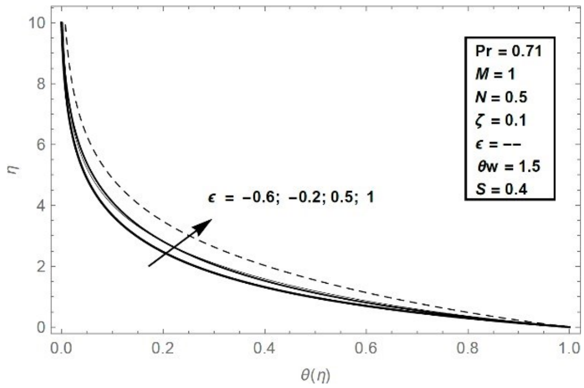

Fig. 4: Temperature graph θ for ε

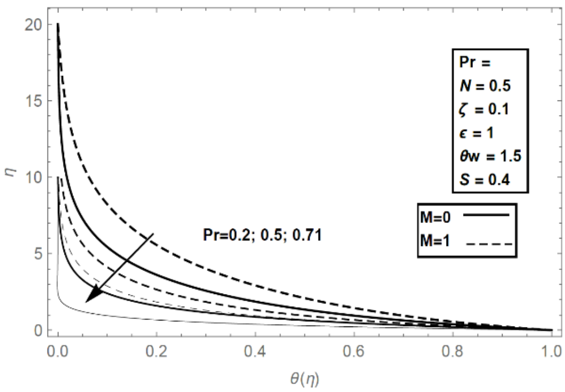

Fig. 5: Temperature graph θ for Pr

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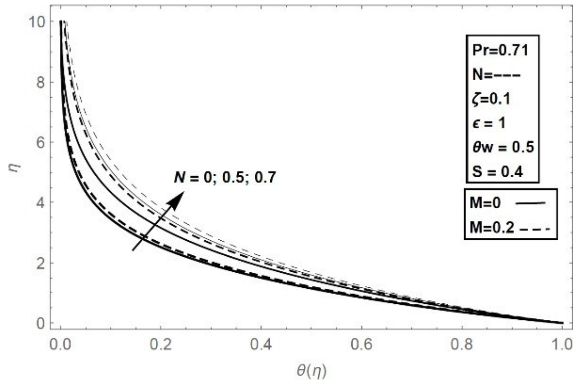

Fig. 6: Temperature graph θ for N

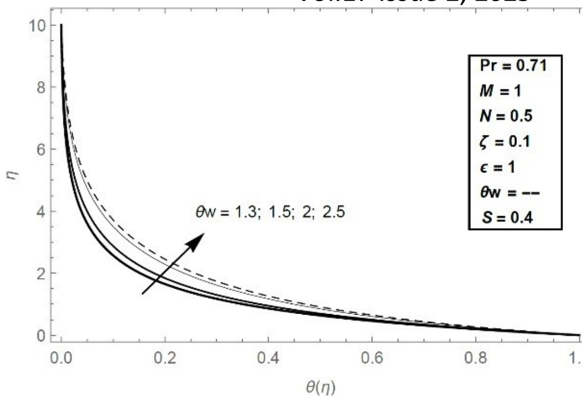

Fig. 7: Temperature graph θ for θ_w

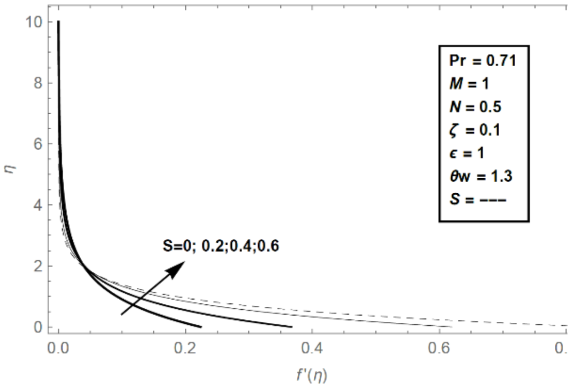

Fig.8a. Velocity graph f' for S

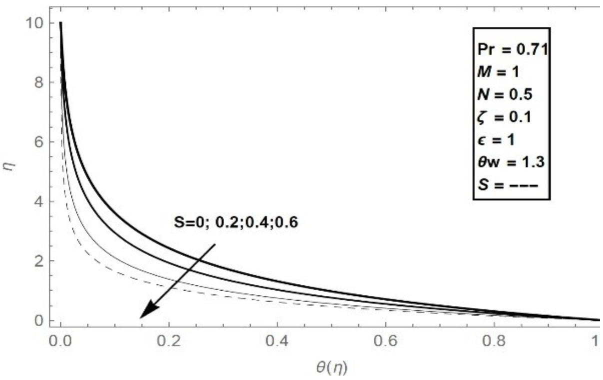

Fig.8b. Temperature graph θ for S

5. Conclusions

The magneto Marangoni fluid flow along an upright circular surface with non-linear pressure work and heat radiation was studied numerically. The effects of the magnetic field, the pressure work and the thermal radiation on the velocity and the temperature profiles are thoroughly examined and illustrated graphically. Additionally, Nusselt number was calculated for the values of different factors which were shown in a tabular form. The key elements of the current results are provided as follows:

- The velocity magnitudes increase for the higher values of buoyancy parameter. In contrast, the opposite effect occurs for both the magnetic parameter and the ratio of the surface tension.
- The temperature values increase for the individual increase in the magnetic field, the thermal radiation heat flux, the surface heating parameter and the pressure work parameter while the temperature values decrease due to the increase in Marangoni number and Prandtl number.

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- Increasing Marangoni ratio number, Prandtl number and the buoyancy parameter enhance Nusselt number. In contrast, the opposite result is obtained for the magnetic field, the pressure work parameter, the radiation parameter and the surface heating parameter.

Table 2: Variation of $-\theta'(0)$ with $M, \xi, \epsilon, Pr, N, \theta$ and S.

	M	ζ	ϵ	Pr	N	θ_w	S	$-\theta'(0)$
M	0.3	0.1	1	0.71	0.5	1.5	0.4	1.2972
	0.5							1.2335
	1							1.0930
ζ	1	0.2	1	0.71	0.5	0.5	1	0.6065
		0.5						0.6228
		0.8						0.6401
ϵ	1	0.1	-0.6	0.71	0.5	0.5	0.4	0.7859
			-0.2					0.6930
			0.5					0.6362
Pr	1	0.1	1	0.2	0.5	1.5	0.4	0.4544
				0.5				0.8582
				0.72				1.0929
N	0.2	0.1	1	0.71	0	0.5	0.4	0.6149
					0.5			0.4849
					0.7			0.4490
θ_w	1	0.1	1	0.71	0.5	1.4	0.4	1.1708
						1.5		1.0930
						2		0.9337
S	1	0.1	1	0.71	0.5	1.3	0.1	0.7561
							0.2	0.9713
							0.4	1.2980

5. Nomenclature

B_0	The constant magnetic field intensity	r, z	coordinates (m)
c_p	specific heat at constant pressure, (J kg^{-1} K^{-1})		**Greek symbols**
f	dimensionless velocity	α	Thermal diffusivity
g	gravity, (m s^{-2})	β	Thermal expansion coefficient
k^*	mean absorption coefficient (m^{-1})	ψ	Stream function
M	magnetic parameter	η	Dimensionless coordinator
N	radiation parameter	θ	Dimensionless temperature
Nu	local Nusselt number	θ_w	Surface heating parameter
P	the variable hydrostatic pressure	μ	Dynamics viscosity of the fluid (N. s/m^2)
P_r	Prandtl number	ν	Kinematic viscosity, (m^2 s^{-1})
q_r	radiative heat flux, (kg m^{-2})	ϵ	Pressure work parameter
Re	local Reynolds number	ρ	Density of the fluid (kgm^{-3})

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S	Surface tension ratio (Marangoni number)	σ	The surface tension at the interface (N/m)
T	Temperature, (K)	σ_1	Stefan-Boltzmann constant
T_w	Surface temperature, (K)	σ_0	Interface surface tension ratio
T_∞	Free temperature, (K)	ξ	Buoyancy parameter
u, w	Velocity components (m s^{-1})		

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