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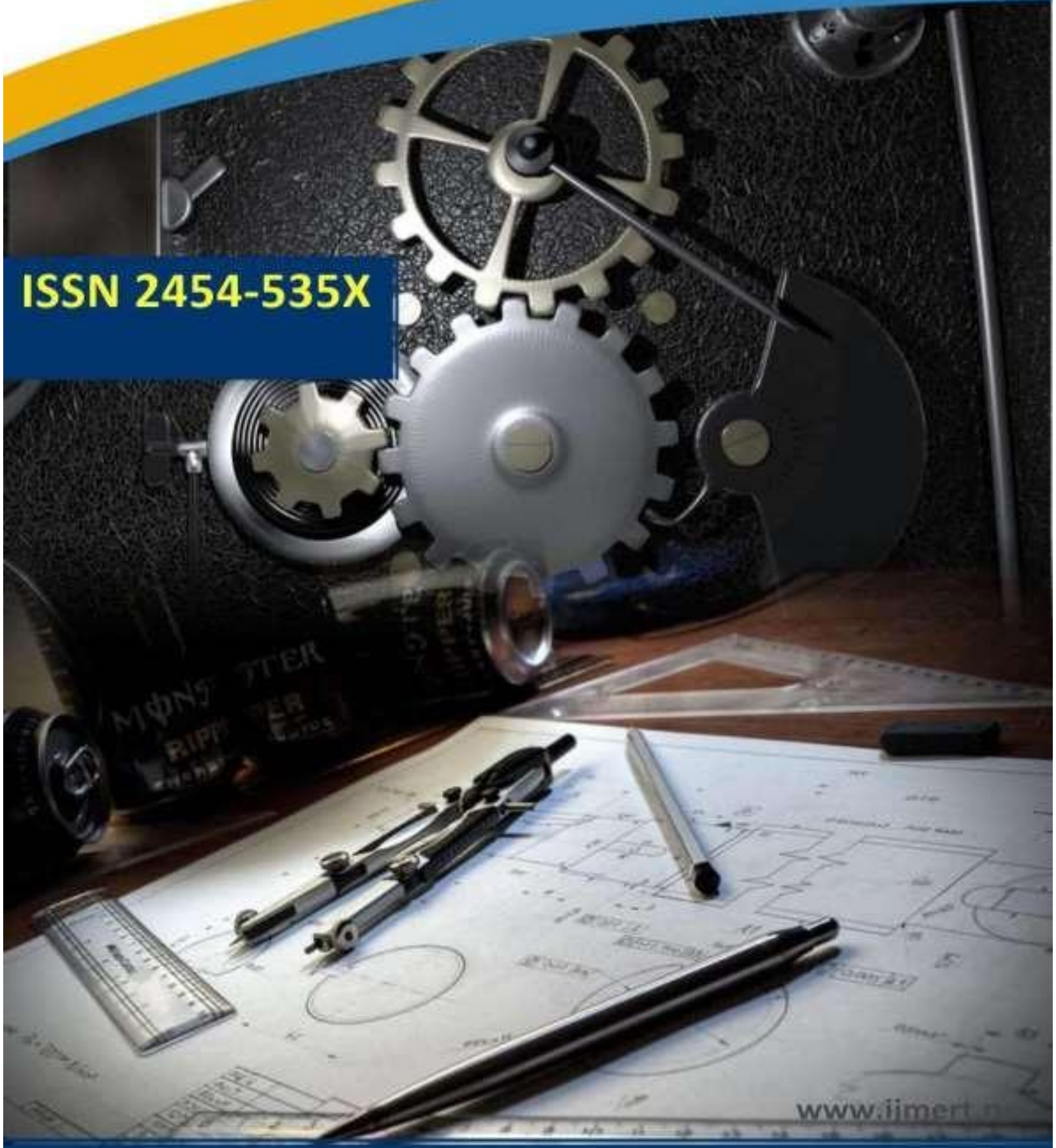

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STRUCTURAL INTEGRITY ASSESSMENT OF A TWO-STAGE DISK WITH PINHOLES UNDER STATIC CONDITIONS

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Abstract

Multistage cyclic symmetry analysis enables efficient simulation of complex rotating structures composed of multiple cyclically symmetric subsystems with differing sector counts. This study presents a static structural assessment of a two-stage rotating disk with stress-relief pinholes using multiharmonic cyclic symmetry analysis in ANSYS Mechanical (Release 2025 R2). The inner stage consists of 11 sectors, while the outer stage contains 16 sectors. Although subjected to axisymmetric centrifugal loading at 100 rad/s, interstage coupling generates non-axisymmetric internal force distributions, requiring inclusion of multiple harmonic indices to ensure displacement continuity and accurate stress prediction. The results demonstrate that single-harmonic (HI = 0) analysis leads to interface discontinuities, while multiharmonic enrichment restores compatibility and produces physically consistent deformation patterns.

Keywords Multistage cyclic symmetry; Harmonic index; Nodal diameter; Rotating disk; Centrifugal loading; ANSYS Mechanical; Structural integrity; Turbomachinery.

1. Introduction

Rotating components such as turbine disks, compressor rotors, and multi-stage rotor assemblies are fundamental to aerospace and power-generation systems. These components frequently exhibit circumferential periodicity, making cyclic symmetry analysis an effective computational strategy.

Traditional cyclic symmetry assumes a single periodic structure under symmetric loading. However, real engineering systems often consist of multiple cyclic subsystems with different sector counts. In such multistage configurations, interstage interactions introduce circumferentially varying internal forces even under axisymmetric loading.

The present study reformulates the ANSYS Mechanical tutorial problem *"Static Analysis of a 2-Stage Disk with Pinholes"*

into a structured technical journal paper. The focus is to demonstrate:

The limitations of single-harmonic cyclic analysis,

The necessity of multiharmonic enrichment,

Proper harmonic index selection based on nodal diameter compatibility,

Validation through path-based interface deformation evaluation.

2. Theoretical Background

2.1 Cyclic Symmetry Formulation

For a cyclic structure with Nidentical sectors, the displacement field can be expressed as a superposition of circumferential harmonics:

$$u(\theta) = \sum_{k=0}^{N-1} U_k \, e^{ik\theta}$$

where:

k= harmonic index (HI),

θ= circumferential angle,

U_k= harmonic displacement component.

For $HI = 0$, deformation is axisymmetric.

2.2 Nodal Diameters

Harmonic indices correspond to nodal diameters (ND), representing circumferential deformation waves. In multistage systems:

Each stage has different allowable harmonic indices.

Only harmonics producing compatible nodal diameters across stages can be effectively coupled.

Improper harmonic selection results in interface discontinuity.

2.3 Multistage Cyclic Symmetry

In the present model:

Stage 1: 11 sectors

Stage 2: 16 sectors

Since 11 and 16 share no common divisor, no single global periodicity exists. Therefore, multiharmonic representation is mandatory for interface compatibility.

3. Materials

In finite element analysis of rotating machinery, the choice of material model plays a critical role in determining both solution accuracy and computational efficiency. For many engineering applications involving disks, rotors, and hubs operating within their elastic limits, the material behavior can be accurately represented using a **linear elastic, isotropic continuum model**. This assumption is widely adopted in classical rotor mechanics and is consistent with the modeling philosophy used in standard ANSYS Mechanical tutorial problems.

The disk material is modeled as:

Homogeneous

Isotropic

Linear elastic

Assumptions:

Small strain

No plasticity

Constant Young's modulus

Constant Poisson's ratio

Constant density

Density is essential for centrifugal loading:

$$F_c = \rho\omega^2 r$$

No thermal or nonlinear material effects are included.

- small strains,
- small rotations (kinematically), and
- no material nonlinearity.

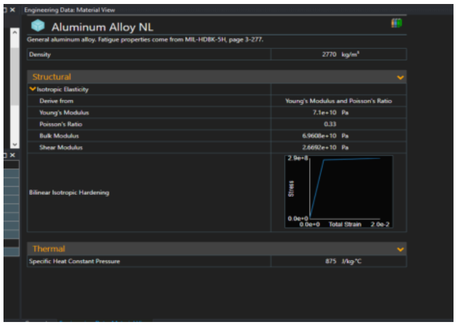
Even though the structure is rotating, the analysis remains linear with respect to material behavior because the rotational loading is applied as a known body force rather than as a kinematic nonlinearity.

Engineering Data: Material View

MAT1

Density	8000 kg/m³
Structural	
Isotropic Elasticity	
Derive from	Young's Modulus and Poisson's Ratio
Young's Modulus	1e+11 Pa
Poisson's Ratio	0.3
Bulk Modulus	8.3333e+10 Pa
Shear Modulus	3.8462e+10 Pa

Stainless steel



Engineering Data: Material View

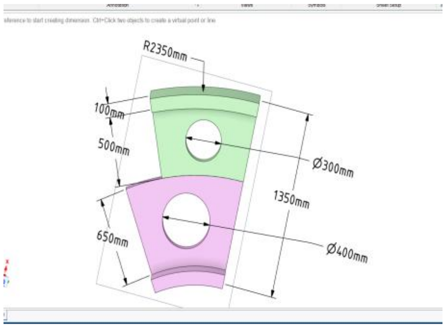
Aluminum Alloy NL

General aluminum alloy. Fatigue properties come from MIL-HDBK-5H, page 3-277.

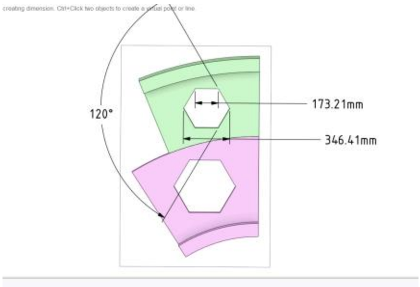
Density	2770 kg/m³
Structural	
Isotropic Elasticity	
Derive from	Young's Modulus and Poisson's Ratio
Young's Modulus	7.1e+10 Pa
Poisson's Ratio	0.33
Bulk Modulus	6.9608e+10 Pa
Shear Modulus	2.6692e+10 Pa
Bilinear Isotropic Hardening	
Thermal	
Specific Heat Constant Pressure	875 J/kg·°C

Aluminum Alloy

4. Geometry and Methodology



Two stage pin holes



Hexagonal shape

4.1 Two-Stage Disk Configuration

The structural model under investigation represents a **two-stage rotating disk assembly** commonly encountered in multi-stage turbomachinery and rotor systems.

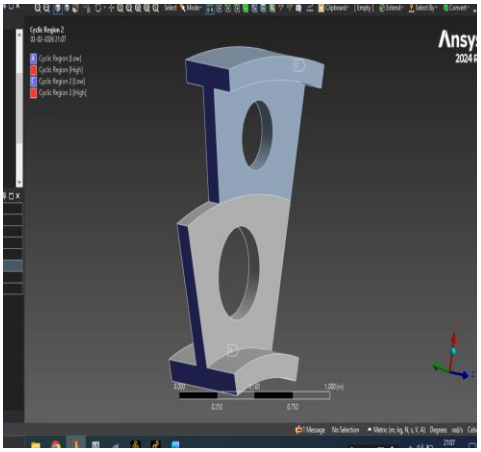
The assembly consists of:

Inner stage (11 sectors)

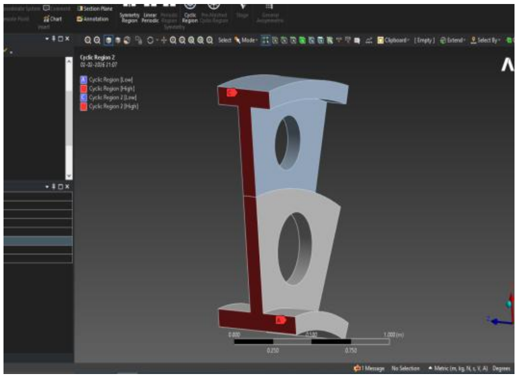
The **inner stage** is located closer to the shaft and represents the portion of the disk that is rigidly constrained at its inner radius. This stage:

- occupies a smaller radial extent,
- includes a periodic array of pinholes distributed uniformly around the circumference,
- is subdivided into **11 identical sectors**, each spanning an angle of

$$\theta_1 = \frac{2\pi}{11}.$$



Cyclic Region



Cyclic Region

Outer stage (16 sectors)

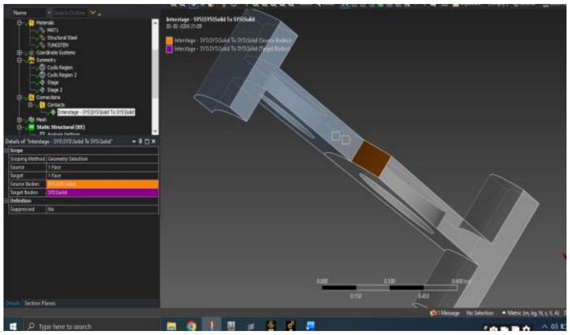
Stress-relief pinholes

Common rotational axis

Only one sector per stage is modeled using cyclic symmetry reduction.

4.2 Stage Definition

Two independent cyclic regions are defined:



Details of "Interstage - SYS\SYS\Solid To SYS\Solid"

Scope	
Scoping Method	Geometry Selection
Source	1 Face
Target	1 Face
Source Bodies	SYS\SYS\Solid
Target Bodies	SYS\Solid
Definition	
Suppressed	No

Inter Stage

Stage 1: 11-sector periodicity

Stage 2: 16-sector periodicity

Interstage coupling is enforced through harmonic-domain constraint equations.

4.3 Interstage Coupling

The interstage connection ensures:

Displacement continuity

Force equilibrium

Because sector counts differ, harmonic superposition is required for compatibility.

5. Meshing Strategy

3D quadratic solid elements used

Automatic mesh generation

Refined mesh near pinholes

Compatible element sizing at interface

Advantages:

Accurate stress gradient capture

Stable harmonic coupling

Efficient DOF count

5.2 Element Type Selection

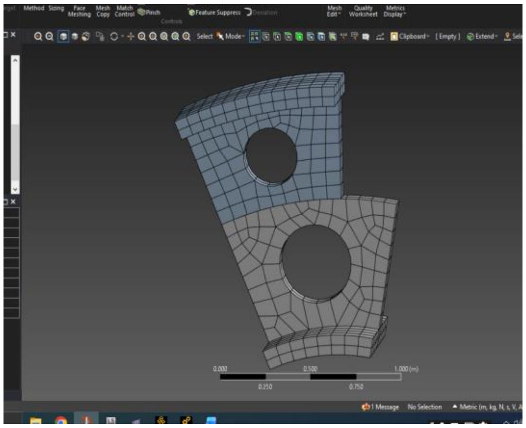
The model uses **three-dimensional quadratic (second-order) solid elements**, which are the standard choice in ANSYS Mechanical for high-accuracy structural analyses. These elements possess mid-side nodes that allow the displacement field within each element to vary quadratically, offering several advantages over linear elements:

- Improved representation of curved geometry, particularly along cylindrical surfaces and pinhole boundaries.
- Enhanced stress accuracy in bending-dominated regions.
- Reduced sensitivity to mesh distortion compared to first-order elements.

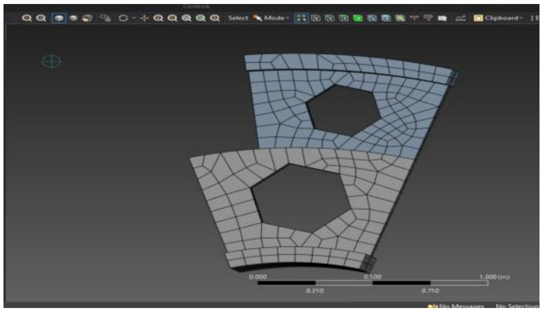
In the context of rotating disks, stress gradients can be significant near:

- pinholes and geometric discontinuities,
- interstage interfaces, and
- constrained boundaries.

Quadratic elements are better suited to capture these gradients without requiring excessively fine meshes. Furthermore, the use of second-order elements improves the accuracy of displacement continuity across cyclic and interstage constraint boundaries, which is critical for multiharmonic coupling.



Mesh



Mesh Hex

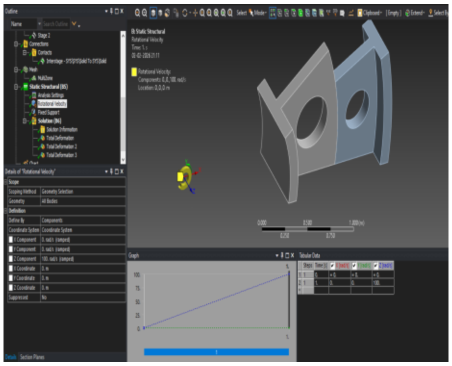
6. Boundary Conditions

6.1 Role of Boundary Conditions in Rotating Multistage Systems

Boundary conditions define how a structure interacts with its surroundings and are essential for obtaining a unique and physically meaningful solution. In rotating machinery, boundary conditions must simultaneously:

- prevent rigid body motion,
- represent realistic mechanical attachment,
- enable correct transmission of centrifugal loads, and
- remain compatible with cyclic symmetry and harmonic decomposition.

In a multistage cyclic symmetry analysis, boundary conditions must also be formulated such that they do not artificially suppress or distort circumferential harmonic content. Improper constraint definition can invalidate the harmonic coupling mechanism that the analysis seeks to capture.

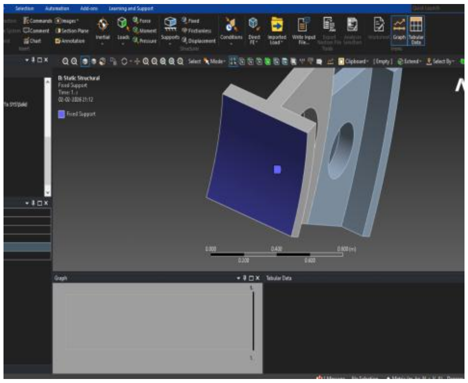


Rotational Velocity

6.1 Fixed Support

Inner radius fully constrained:

$$u_r = u_\theta = u_z = 0$$



Fixed Support

6.2 Rotational Loading

Angular velocity:

$$\omega = 100 \text{ rad/s}$$

For single-harmonic analysis:

Mechanical Rotational Velocity used.

For multiharmonic analysis:

APDL command CMOMEGA applied.

6.3 Cyclic Boundary Conditions

Harmonic phase relation:

$$u_{high} = u_{low} e^{ik\alpha}$$

where α= sector angle.

7. Results and Discussion

7.1 Single Harmonic (HI = 0)

Observations:

Global deformation appears smooth.

Path-based interface evaluation shows displacement mismatch.

Interface discontinuity confirms missing harmonic content.

Conclusion: Axisymmetric assumption insufficient.

7.2 Multiharmonic Analysis (HI = 0, 5)

After including harmonic index 5:

Interface displacement continuity restored.

Path-based curves overlap.

Physically realistic deformation obtained.

7.3 Material and Shape Comparison

Circular Geometry

Material	Deformation (m)
Structural Steel	0.00253
Aluminum	0.00121

Hexagonal Geometry

Material	Deformation (m)
Structural Steel	0.00265
Aluminum	0.00127

Multiharmonic case (HI = 0, 5) circular deformation:

0.00166 m

Aluminum exhibits lower deformation due to reduced density.

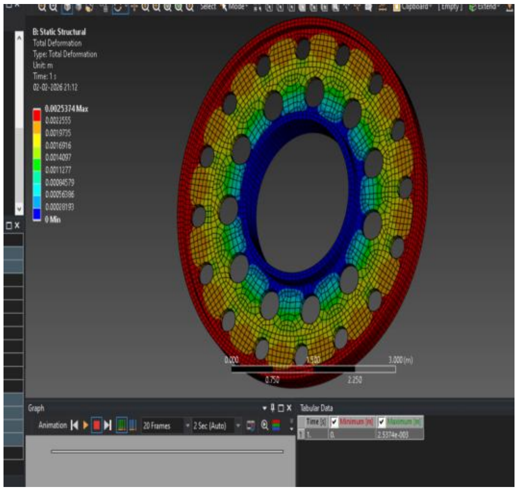
7.4 Key Observations

Single-harmonic cyclic analysis is incomplete.

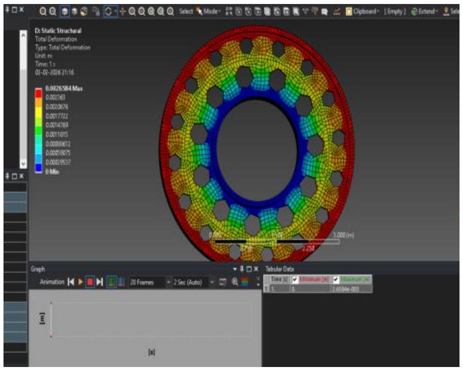
Multiharmonic enrichment restores compatibility.

Harmonic selection must be based on nodal diameter theory.

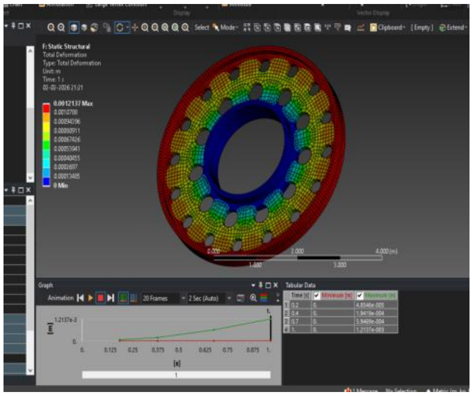
Path-based interface evaluation is essential.



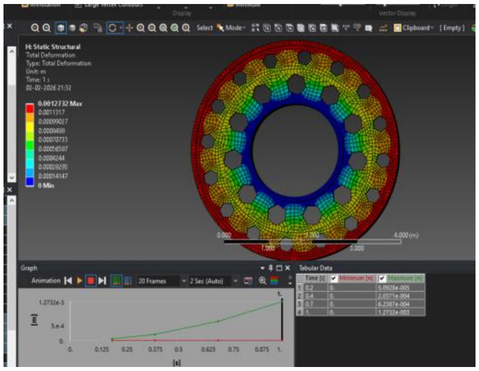
Deformation



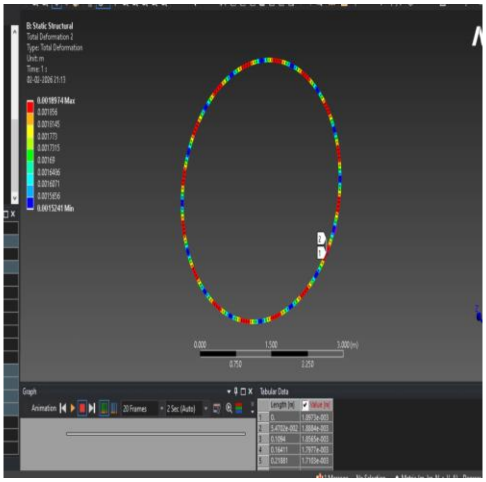
Deformation hex



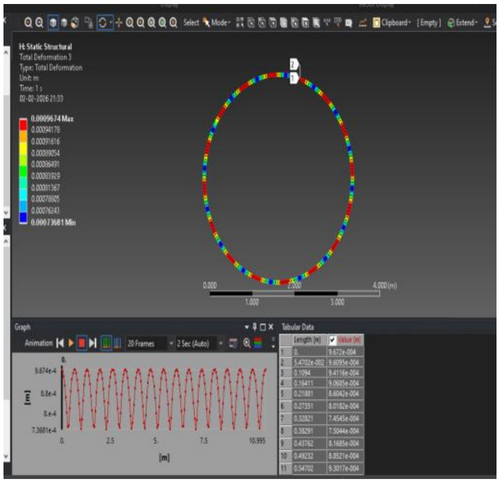
Deformation aluminum



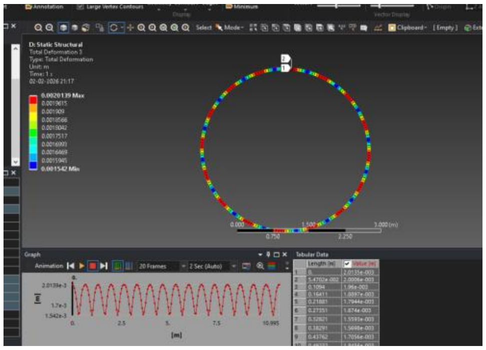
Deformation aluminum hex



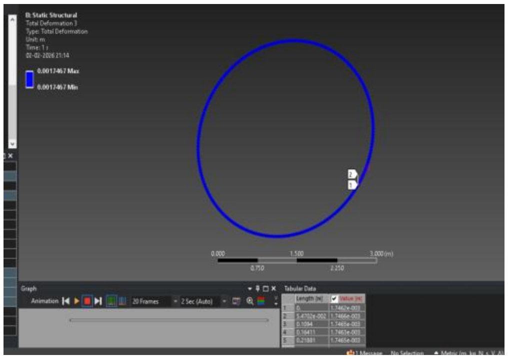
Deformation path



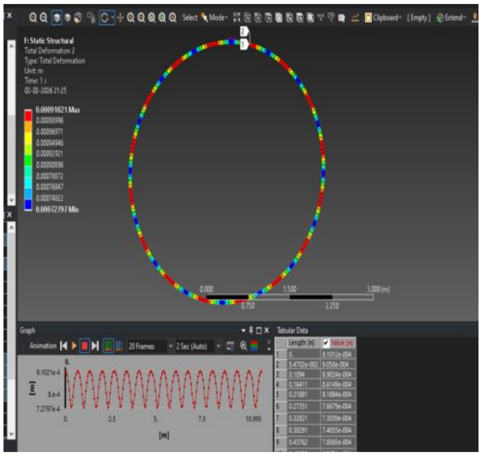
Deformation path aluminum hex



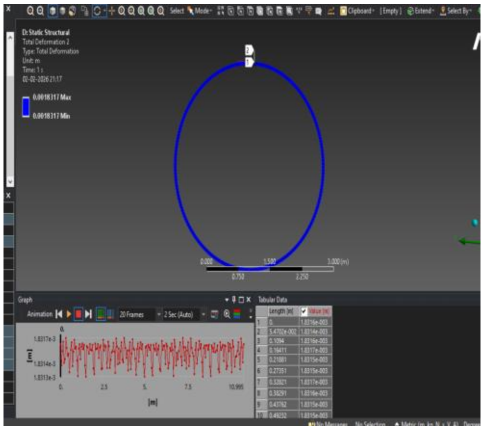
Deformation path hex



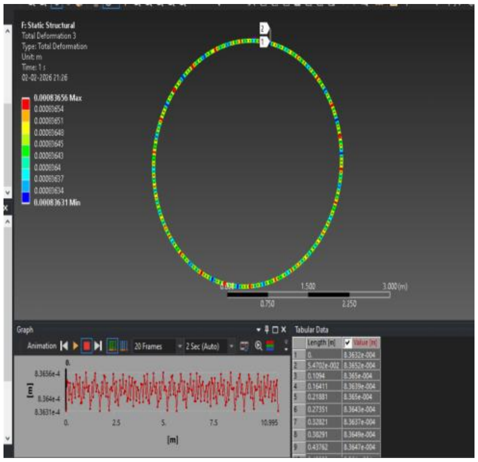
Deformation path 2



Deformation path aluminum

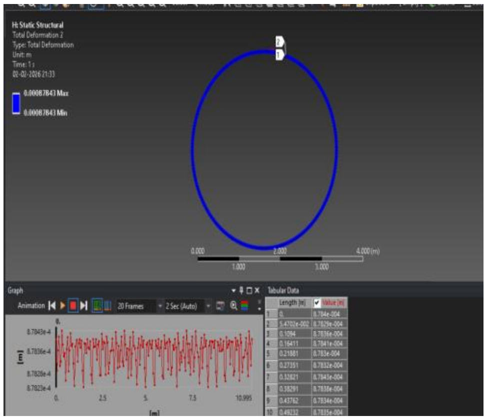


Deformation path 2 hex



	Length [m]	Value [m]
1	0.	8.3632e-004
2	5.4702e-002	8.3652e-004
3	0.1094	8.365e-004
4	0.16411	8.3639e-004
5	0.21881	8.365e-004
6	0.27351	8.3643e-004
7	0.32821	8.3637e-004
8	0.38291	8.3649e-004
9	0.43762	8.3647e-004

Deformation path2 aluminum



	Length [m]	Value [m]
1	0.	8.784e-004
2	5.4702e-002	8.7829e-004
3	0.1094	8.7836e-004
4	0.16411	8.7841e-004
5	0.21881	8.783e-004
6	0.27351	8.7832e-004
7	0.32821	8.7843e-004
8	0.38291	8.7838e-004
9	0.43762	8.7834e-004
10	0.49232	8.7835e-004

Deformation path2 aluminum hex

8. Conclusion

This study demonstrates the effective extension of **multistage cyclic symmetry analysis** to **multiharmonic static problems** in rotating structures, using a two-stage disk with stress-relief pinholes as a representative example. The example highlights both the **limitations of conventional single-harmonic cyclic modeling** and the **significant benefits of a carefully constructed multiharmonic formulation** when analyzing coupled rotating components with different circumferential periodicities

Final Remarks

In conclusion, this example confirms that:

- **Multistage cyclic symmetry** is a robust and efficient framework for modeling coupled rotating components with different periodicities.
- **Multiharmonic enrichment** is essential whenever interstage coupling excites non-axisymmetric deformation modes.
- **Nodal-diameter-based harmonic selection** is the theoretical foundation for accurate and stable solutions.

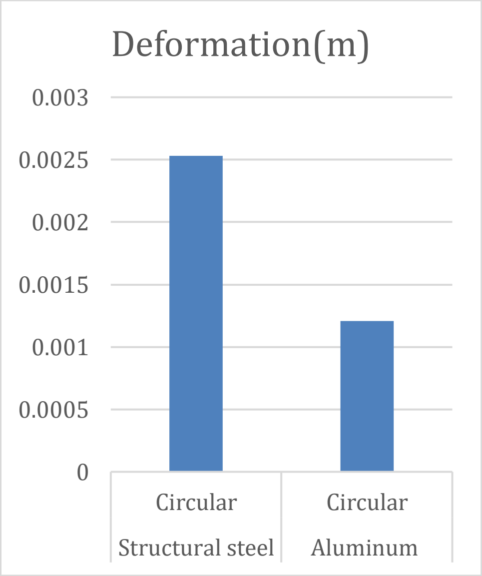
- **Path-based interface evaluation** provides a clear and practical method for validating solution adequacy.

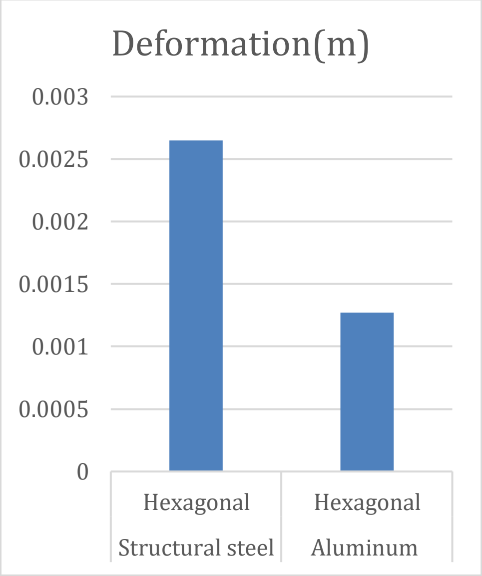
When applied correctly, this approach enables engineers to achieve **high-fidelity structural predictions** while maintaining manageable computational cost, making it an indispensable tool in advanced rotating machinery analysis.

S. No	Material	Shape	Deformation(m)
1	Structural steel	Circular	0.00253
2	Aluminum	Circular	0.00121

S. No	Material	Shape	Deformation(m)
1	Structural steel	Hexagonal	0.00265
2	Aluminum	Hexagonal	0.00127

Harmonic index 0,5 circular deformation 0.00166m.





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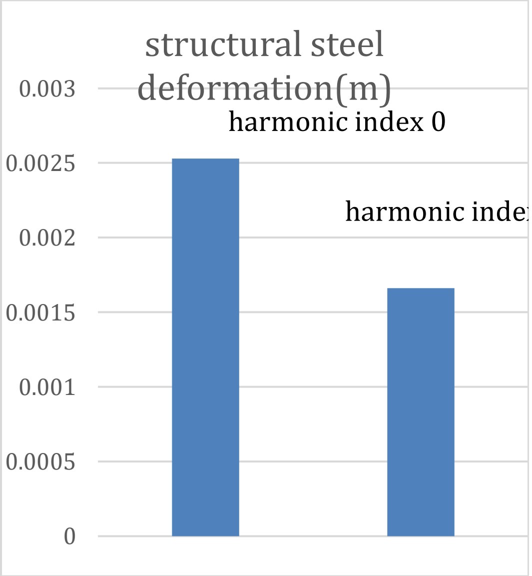

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