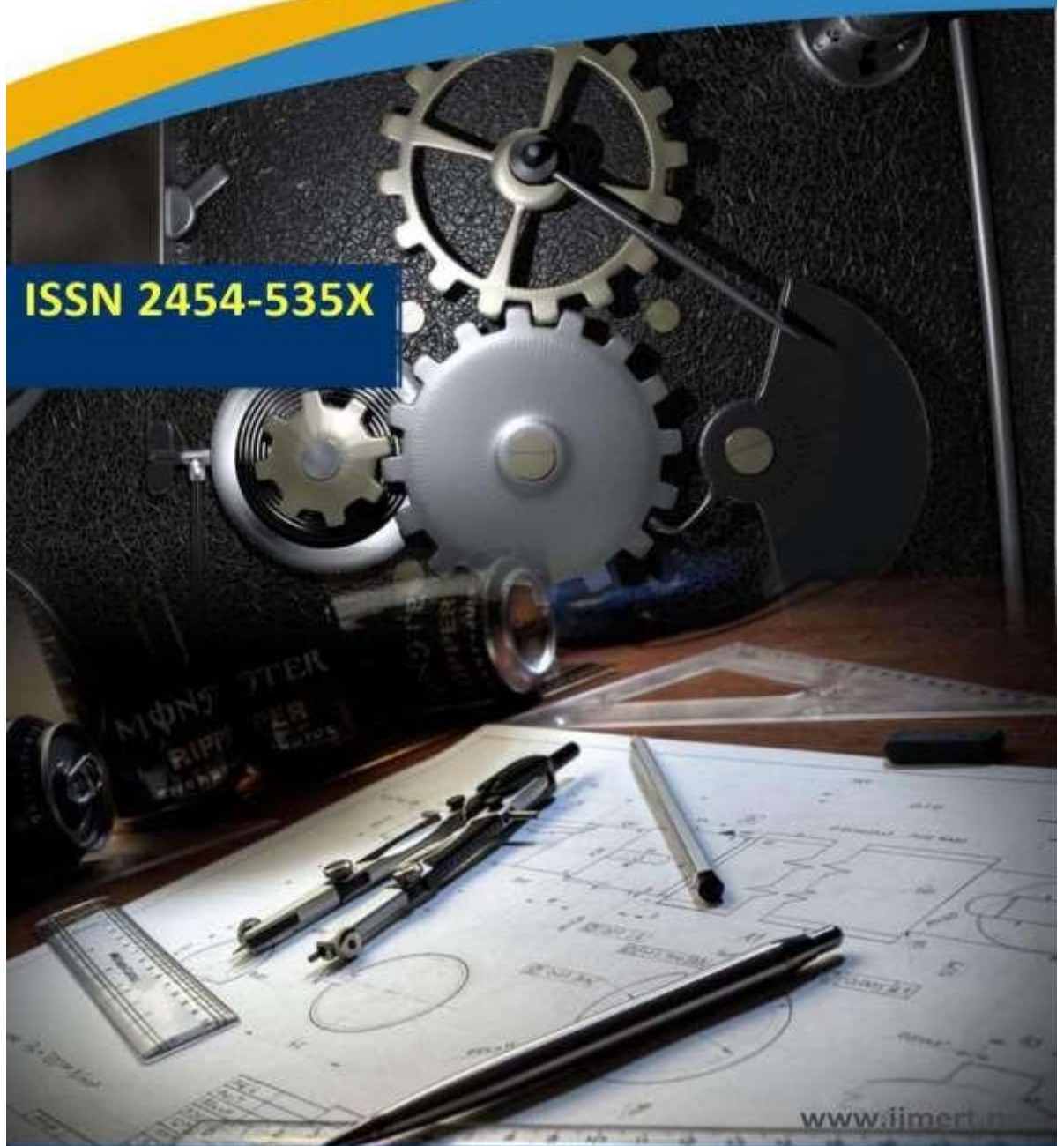




International Journal of
Mechanical Engineering Research and Technology

ISSN 2454-535X



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Analysis of Composite Plate Failure Under Arbitrary Tension Load with Central Opening Hole

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ABSTRACT:

The most feasible composite construction has holes built into its basic structure; these holes result in strong stress gradients. In the construction of buildings, it is crucial to comprehend how the cutout affects the failure criterion of such composite plates. The failure loads surrounding a hole in an E-glass/Epoxy composite plate are investigated in this study under the influence of numerous variables, including fiber volume percentage, loading angle, aspect ratio, and fiber orientation. Two methods are employed to accomplish the goal. I- Analytical, Lekhnitskii's solution was implemented using MATLAB version 17, and the generalized formulation that was thus obtained is coded to produce analytical results. The simulation model was created by II-Numerical using the finite element program ANSYS 17.2.

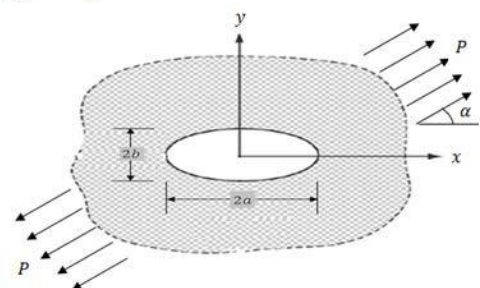
KEYWORDS: Strength Parameter; Failure Index; Fiber Orientation; Aspect Ratio; Fiber Volume Fraction; Loading Angle

INTRODUCTION

Due to their high strength-to-weight ratios, low coefficients of thermal expansion, resistance to fatigue and corrosion, and the possibility to specifically customize the fiber orientations to agiven geometry, applied load, and environment, fiber reinforced composite materials are exceptional in application

[1].To lighten the weight of the structure or to provide access to other portions of the structure, different hole shapes are required in structural elements. Some structural components are failing during the course of their useful lives. It is common knowledge that a cutout or hole in a stressed element causes a highly localized load to be created nearby [2]. Stress concentration factor

$$\left. \begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} &= 0 \end{aligned} \right\} \quad (1)$$



(SCF) values can be calculated experimentally by utilizing the photoelastic method or

THEORETICAL BACKGROUND

An analytical method uses various mathematical expressions to predict the stress analysis of a composite plate with cutout shapes in structure element [8].

Lekhnitskii's Solution for Composite Materials

Consider an infinite anisotropic plane subjected to a uniform traction load, P with



an angle α from the x-axis as shown in Fig.(1). Equilibrium equations into two

dimensional for non – attendance of the body force can be expressed as;

Figure 1. An infinite composite containing a normal elliptical opening The strain

compatibility equations can be expressed as;

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = 2 \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} \quad (2)$$

For the plane stress condition, the following stresses and strains are zero

$$\sigma_{zz} = \sigma_{xz} = \sigma_{yz} = 0, \epsilon_{xz} = \epsilon_{yz} = 0$$

Consider the generalized Hooke's law in an anisotropic material, the in-plane strains can be expressed as

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad (3)$$

Where, a_{ij} is the elastic compliance. The Airy stress function can be expressed as [9]

$$\begin{aligned} \sigma_x &= \frac{\partial^2 \phi}{\partial y^2} \\ \sigma_y &= \frac{\partial^2 \phi}{\partial x^2} \\ \tau_{xy} &= -\frac{\partial^2 \phi}{\partial x \partial y} \end{aligned} \quad (4)$$

Where, ϕ is a potential function that can be solved via boundary condition. Substituting Eq.(4) into Eq.(3) yield

$$\begin{aligned} & \frac{\partial^4 \phi}{\partial y^4} + (2a_{12} + a_{66}) \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + a_{22} \frac{\partial^4 \phi}{\partial x^4} - 2a_{16} \frac{\partial^4 \phi}{\partial x \partial y^3} - 2a_{26} \frac{\partial^4 \phi}{\partial y \partial x^3} = 0 \quad (5) \end{aligned}$$

The general solution of Eq.(5) in the form $\phi(x, y) = F(x + sy)$, where s is a complex variable.

Therefore Eq.(5) becomes a characteristic equation expressed as;

$$a_{11}s^4 - 2a_{16}s^3 + (2a_{12} + a_{66})s^2 - 2a_{26}s + a_{22} = 0 \quad (6)$$

The roots of this characteristic equation are complex so $s_{1,3} = \alpha_1 \mp i\beta$, $s_{2,4} = \alpha_2 \mp i\beta$,

The general solution of ϕ is given by;

$$\phi(x, y) = F_1(x + s_1 y) + F_2(x + s_2 y) + F_3(x + \bar{s}_1 y) + F_4(x + \bar{s}_2 y) \quad (7)$$

Because of ϕ is real Eq. (7) can be written as

$$\phi(x, y) = 2\text{Re}[F_1(z_1) + F_2(z_2)] \quad (8)$$

Where, $z_{jj} = x + s_{jj}y$, $jj = 1, 2$

By solving the characteristic equation and applying the boundary conditions, the in-plane stress can be expressed as follows [10];

$$\begin{aligned} \sigma_x &= 2\text{Re} \left[\frac{1}{s^2} + s^2 \right] \\ \sigma_y &= 2\text{Re} \left[\frac{1}{s^2} + s^2 \right] \end{aligned}$$



$$\begin{aligned}\sigma_y &= 2\operatorname{Re}[\Phi_1 + \Phi_2] \\ r_{xy} &= -2\operatorname{Re}[s_1\Phi_1 + s_2\Phi_2]\end{aligned}\quad (9)$$

Savin's Solution for Stress in Composite

Since it is traction free on the surface around the hole as shown in Fig. (1), we can express the traction are;

$$\begin{aligned}f_1 &= Y Y_m ds = 2\operatorname{Re}[\Phi_1 + \Phi_2] = 0 \\ f_2 &= X Y_m = -2\operatorname{Re}[s_1\Phi_1 + s_2\Phi_2] = 0\end{aligned}\quad (10)$$

The potential function Φ_1 and Φ_2 obtained from Savin's solution can be expressed as [10];

$$\begin{aligned}\Phi_1(z_1) &= B_1 z_1 + \psi_0(z_1) \\ \Phi_2(z_2) &= \left(\frac{B_2 + iC_1}{2}\right) z_2 + \psi_0(z_2)\end{aligned}\quad (11)$$

The coefficient can be solved as;

$$\beta_1 = \frac{\cos^2\alpha + (\cos^2\alpha)\sin^2\alpha + \alpha_2^2\sin 2\alpha}{2((\alpha_1^2 - \alpha_2^2)^2 + (\beta_1^2 - \beta_2^2))}$$

$$\beta_2 = \frac{(\cos^2\alpha - 2\alpha_1\alpha_2)\sin^2\alpha - \cos^2\alpha + \alpha_2^2\sin 2\alpha}{2((\alpha_1^2 - \alpha_2^2)^2 + (\beta_1^2 - \beta_2^2))}$$

$$C_1 = \frac{P((\alpha_1 - \alpha_2)\cos^2\alpha + (\alpha_2^2(\alpha_1^2 - \beta_1^2) - \alpha_1(\alpha_2^2 - \beta_2^2))\sin^2\alpha + ((\alpha_1^2 - \beta_1^2) - (\alpha_2^2 - \beta_2^2))\sin\alpha\cos\alpha}{2\beta_1((\alpha_1^2 - \alpha_2^2)^2 + (\beta_1^2 - \beta_2^2))}$$

Where

$$\psi_0(z_1) = -iP(a - is_1b)$$

$$\begin{aligned}\psi_0(z_1 + \sqrt{z^2 - (a^2 - s^2b^2)}) &= \frac{4(s_1 - s_2)b(s_2\sin 2\alpha + 2\cos^2\alpha)}{ia(2s_2\sin^2\alpha + \sin 2\alpha)} \\ \psi_0(z_1 + \sqrt{z^2 - (a^2 - s^2b^2)}) &= \frac{1}{4s_1 - s_2z_2 + \sqrt{z_2^2 - (a^2 - s_2b^2)}} \frac{1}{2} \frac{1}{\sqrt{z_2^2 - (a^2 - s_2b^2)}} \frac{1}{2} \frac{1}{\sqrt{z_2^2 - (a^2 - s_2b^2)}} P(a - 2b)(s_1\sin 2\alpha + 2\cos^2\alpha)\end{aligned}$$

Substituting in Eq.(10) yield, the stress state at each point in the composite plate due to the remote stresses can be expressed as:

$$\begin{aligned}\sigma_x &= \sigma^\infty + 2\operatorname{Re}[s_0^2\Phi_1(z) + s_0^2\Phi_2(z)] \\ \sigma_y &= \sigma^\infty + 2\operatorname{Re}[\Phi_1(z) + \Phi_2(z)] \\ r_{xy} &= r_{xy}^\infty - 2\operatorname{Re}[s_1\psi_0(z_1) + s_2\psi_0(z_2)]\end{aligned}\quad (12)$$

Where, σ_x^∞ , σ_y^∞ and r_{xy}^∞ are in- plane stresses under uniform tension (P) acting at any angle (α) measured from the horizontal axis, $\sigma_x^\infty = P\cos^2\alpha$, $\sigma_y^\infty = P\sin^2\alpha$, $r_{xy}^\infty = P\sin\alpha\cos\alpha$. Therefore, the stress field around

an elliptical hole in an anisotropic plate subjected to stresses and determined by Eq.(12). The transformation of orientated stress to principal stresses is performed by the following matrix expression [11];

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} m^2 & -n^2 \\ 2mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ r_{xy} \end{bmatrix}$$

Where, m and n are cosine and sine of fiber orientation respectively.



Failure Criteria

The strength of a composite lamina may be characterized by five different material parameters. The Tsai-hill criterion is a phenomenological material failure theory, which is widely used for composite materials which have different strength in tension and compression. They are able to predict overall failure, but cannot predict [12];

the exact failure mode. The Tsai-hill criterion predicts failure when the failure index in ply reaches 1. The prediction of the failure of structural components is usually accomplished by comparing the stresses or strains to the Material ultimate limits and this criterion may be expressed in the form

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_{66}\tau^2 \geq 1 \quad (14)$$

Where, $F_1 = \frac{1}{\sigma_{1t}} - \frac{1}{\sigma_{1c}}$, $F_2 = \frac{1}{\sigma_{2t}} - \frac{1}{\sigma_{2c}}$, $F_{11} = \frac{1}{\sigma_{1t}^2} - \frac{1}{\sigma_{1c}^2}$, $F_{22} = \frac{1}{\sigma_{2t}^2} - \frac{1}{\sigma_{2c}^2}$, $F_{66} = \frac{1}{S^2}$, $F_{12} = 0.5 \left(\frac{1}{\sigma_{1t}\sigma_{2t}} - \frac{1}{\sigma_{1c}\sigma_{2c}} \right)$ And, σ_{1t} , σ_{1c} , σ_{2t} , σ_{2c} and S are the tensile strengths of the material in the fiber and transverse directions, respectively and S is the shear modulus. Generally, these expressions are based on the process of adjusting an expression to a curve obtained by experimental tests in next section.

NUMERICAL ANALYSIS

The finite element technique is a powerful tool which can be used in many engineering applications by taking advantage of the continuously developing digital computers with fast capability. The commercial software Ansys 17.2 is user-friendly and easy to design the required model [1, 13]. The meshing process has been done by choosing the suitable element SHELL181 as shown in Fig.(2) with six degree of freedom at each node. A convergence study was carried out to determine how used mesh is suitable for accurate results and computing time during analysis. The location of the elements and therefore the nodes must also reflect any changes in material properties, geometry, constraint conditions and applied loads.

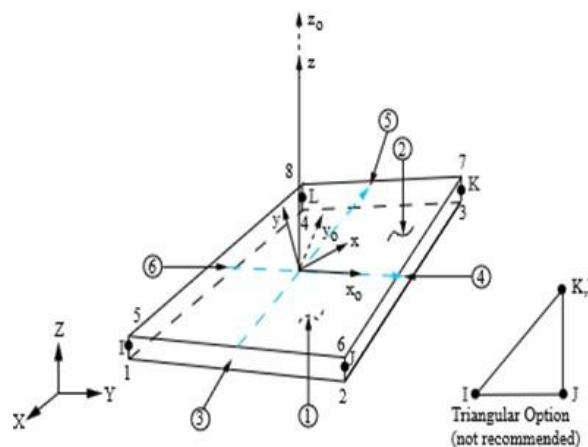


Figure 2. Shell181 element geometry

In Ansys program, due to the symmetric nature of the plate, only quarter models of composite plate with hole and simply supported (S.S.) boundary condition is presented in Fig. (3), where u

and v is the displacement in x

and y direction respectively. The geometry of a plate with various holes can be drawn relatively easily in Ansys program through Design Modeler.

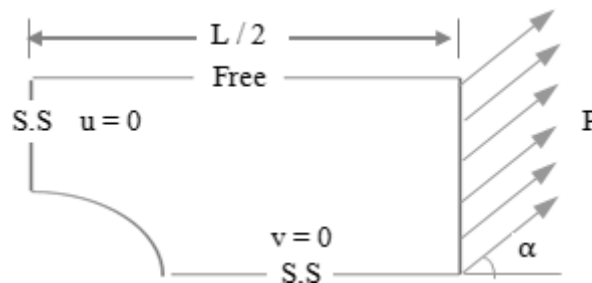


Figure 3. Quarter composite plat with hole

PROBLEM DESCRIPTON

The physical structure that was used in this work is a fiber reinforced composite plate and the material properties are shown in Table.1. A square plate of uniform thickness with an opening central hole subjected to the uniform a constant

pressure $P=10$ MPa at an angle measured from the horizontal axis is considered. The model is analyzed for the different aspect ratio of the elliptical hole by keeping the dimension of the plate fixed. The width of 20 mm and thickness of the plate is 1 mm [14].

Table 1. Material properties of composite plate

Elastic properties	Fiber material (E-glass)	Matrix material (Epoxy)
Young's modulus, GPa	85.0	3.40
Poisson's ratio	0.20	0.30
Shear modulus, GPa	35.42	1.308

EXPERIMENTAL WORK

In the first stage, the majority of the experimental research to find the properties of composite plate by testing the specimen accordance to ASTM standards with five specimens prepared for each test conditions. This test was done by using the material testing machine as shown in Fig. (4). The test was carried out, the tensile strength was found by testing the

specimen for tension test using ASTM D3039 standard. To ensure the accuracy of the experimental results, some tests were prepared at the same conditions. The sample is fix with strain gages in the linear and accidental direction. The loads are applied to the sample at a rate of 4 mm/min. A total of 30 to 40 data points for stress and strain is taken until the fiber starts

to fracture. The step for obtainment accidental tensile strength is such as for finding the linear tensile strength just the sample dimension differs. The compression modulus and strength was obtainment with the combined loading compression test according to ASTM

D3410. The shear modulus and shear strength were found in accordance to ASTM 7979. Finally, the experimental data is summarized in Table.2 for various fiber volume fraction and input into the Ansys program through engineering data [15-18].

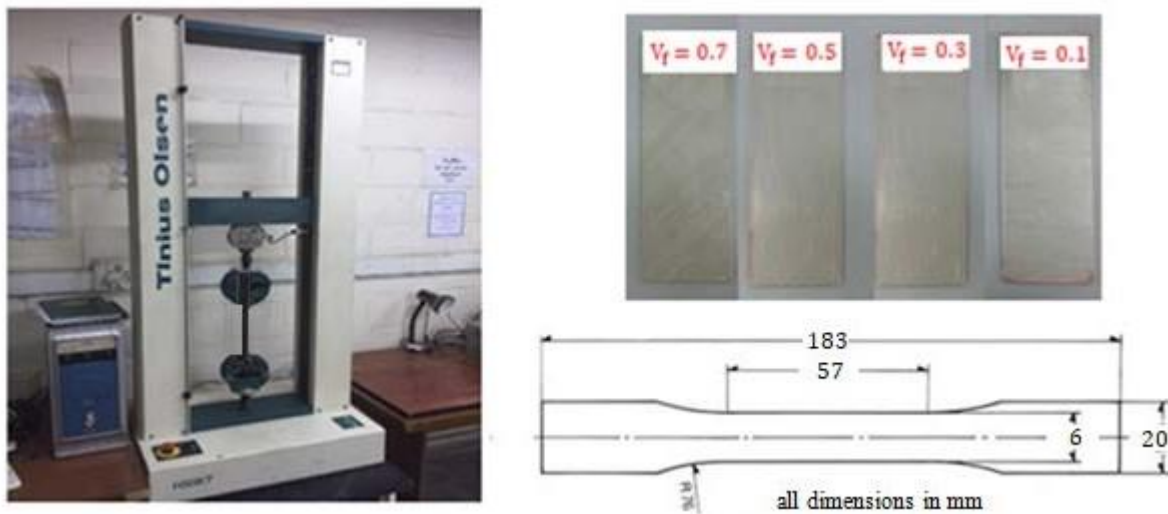


Figure 4. Machine and samples of tensile test for composite material

Table 2. Strength properties for composite ply at different fiber volume fraction

Fiber volume fraction, v_f	Longitudinal tensile strength, X^t (MPa)	Longitudinal compressive strength, X^c (MPa)	Transverse tensile strength, Y^t (MPa)	Transverse compressive strength, Y^c (MPa)	In-plane shear strength, S (MPa)
0.1	210.8	554.9	523.5	741.1	246.9
0.3	508.4	888.9	411.1	582.3	193.4
0.5	806.1	875.4	323.7	458.6	151.7
0.7	1103.6	520.8	205.1	290.6	94.4

RESULTS AND DISCUSSION

Results of the previously described analysis are presented in this section. The analytic method was implemented to research the influence of a variable factor on the failure index for the composite plate. The simulation was carried out using programs that have been written using Matlab code, in order to efficiently

analyze the composite plate.

Figure 5 shows counter of stress fields under tension load (10 MPa) in x-direction at zero loading angle. Herewe can see that the maximum stress is 27.591 MPa occur at the edge of the hole at periphery of hole.

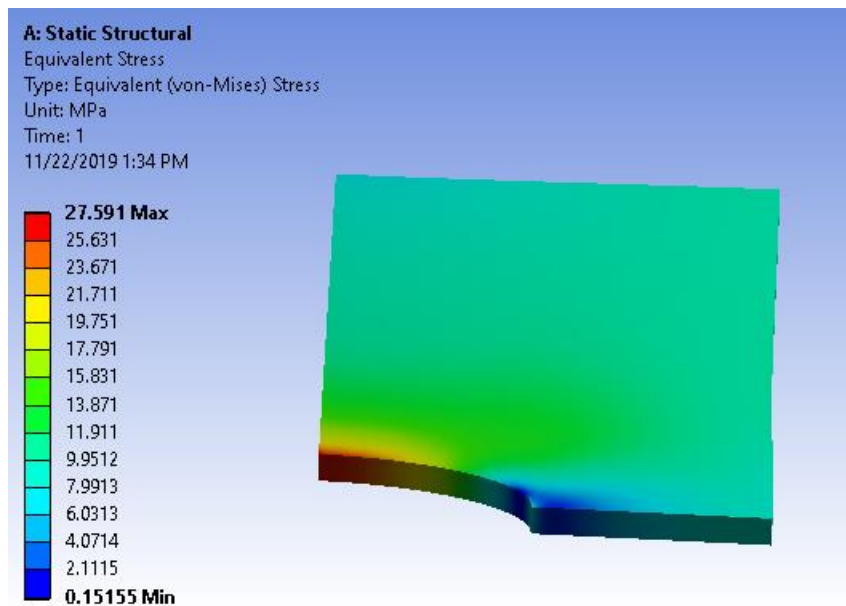
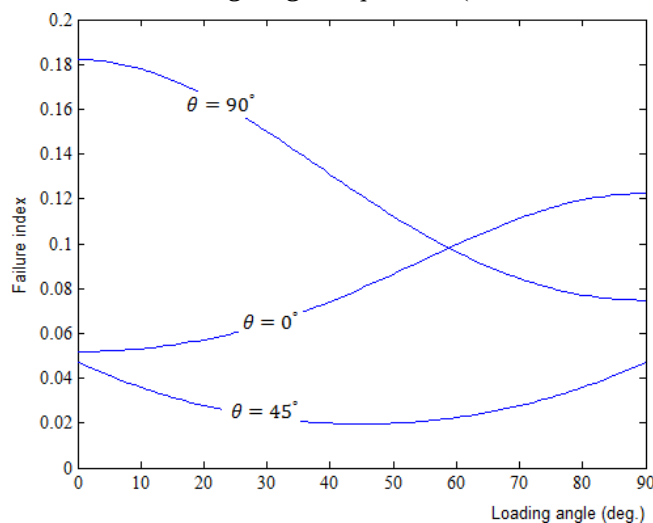


Figure 5. Distribution of max. stress in plate with hole ($b = 1$, $a = 3$) at $vff = 0.2$ and $\theta = 0^\circ$

The value of the failure index of the composite plate is predicted by using Eq. (14). Based on result presented herein, the effect of loading angle on the failure index at different fiber volume fraction and fiber orientation as shown in figures below. According to Fig. (6) as the loading angle increases in this region, the failure index actually slightly increases in all other fiber volumes fraction, but the rate of this increase is not constant, due to strength are most sensitive to material properties. The maximum value of failure index occurred at loading angle equal to ($\alpha =$

90°), whereas the minimum value of a failure index appears to occur at a loading angle equal to ($\alpha = 0^\circ$).

From Fig.(7) it can be seen that ,when the fiber angle equal to the maximum value ($\theta = 90^\circ$), the failure index smoothly decrease with the increase of the loading angle, due to stress ratio is most sensitive to material properties and directly depend on the ratio of moduli's. Conversely when the fiber angle equal to the minimum value($\theta = 0^\circ$), the fibers are parallel to the x-axis, the failure index increase with the increase of the loading angle. Another important point in these figures is observed that, when the fiber angle equals to ($\theta = 45^\circ$), the failure index smoothly decrease with the increase of the loading angle, until the value of loading angle equals to ($\alpha = 45^\circ$) and after that, the failure index slowly increase with the increase of the loading angle.



The effect of hole size on the failure index predicted at different fiber volume fraction and fiber angle as shown in Figs. (8 and 9) respectively. It is observed that as the aspect ratio a / b increases the failure index reduced gradually and the curves



become nearly flat. It is shown that from the figures, the rise in failure index in the vicinity of the hole is quite sharp as compared to the circular hole. In general, it can be concluded that the failure index is an approximately constant value when the aspect ratio greater than unity.

Figures (10) and (11) show that analytical results are in good agreement with finite element method using Ansys program, the discrepancy results obtained is less than 5 %. In general, the Ansys program

gives a better approximation of failure behavior when the major diameter of the hole is small. But as the diameter is increased, the deviation of the Ansys results from the analytical one is higher and is equal to 12 %. Moreover, the deviation increases with loading angle. One reason for the deviation is that the approximate solutions and it's to avoid the discontinuities of stress that occur across inters element boundaries in compatible finite element method.

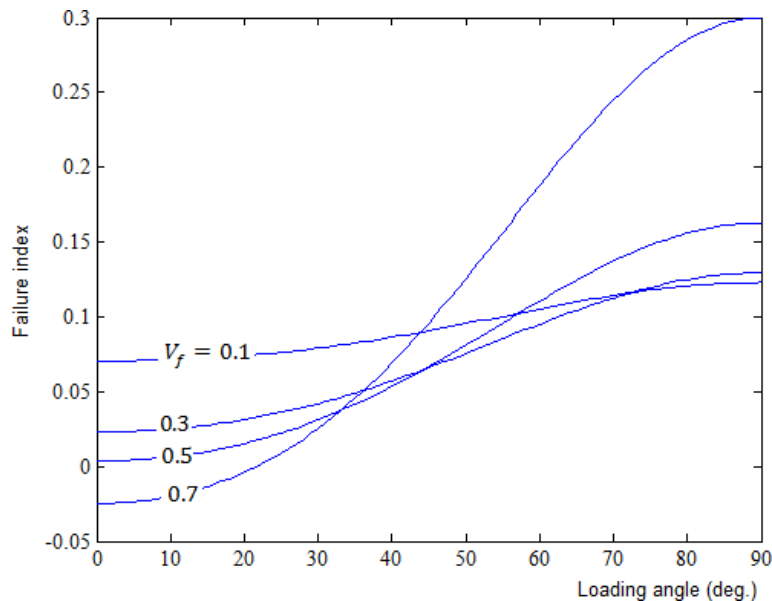


Figure 6. Failure index with loading angles at different fiber volume fraction of composite

Figure 7. Failure index with loading angles at different fiber orientation of composite

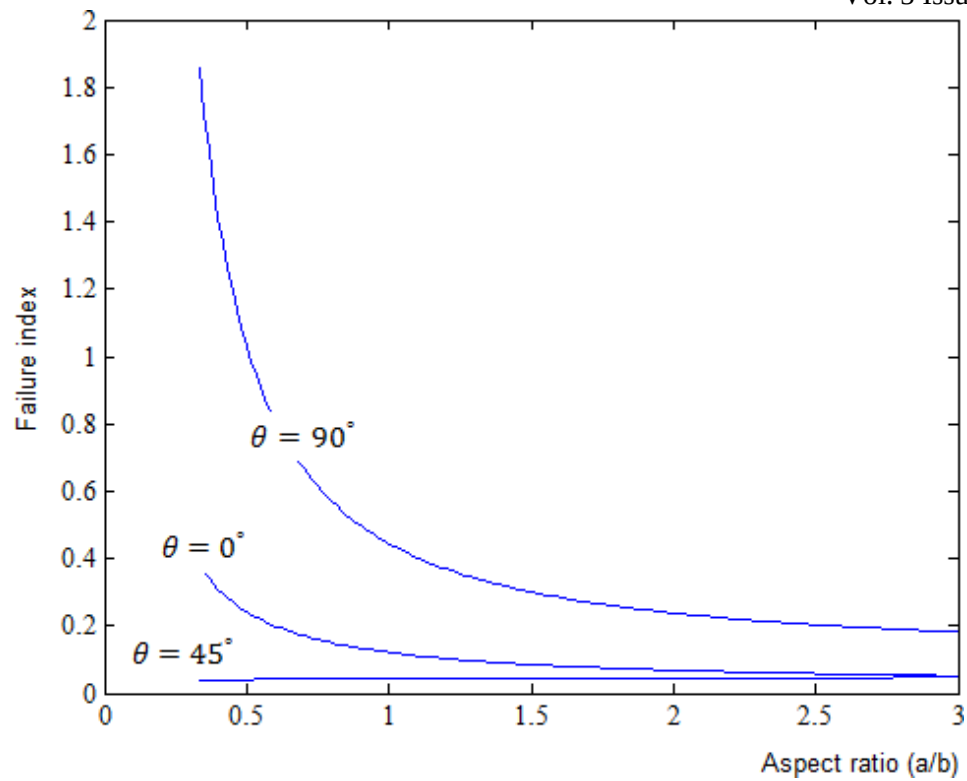


Figure 8. Failure index with aspect ratio at different fiber orientation of composite

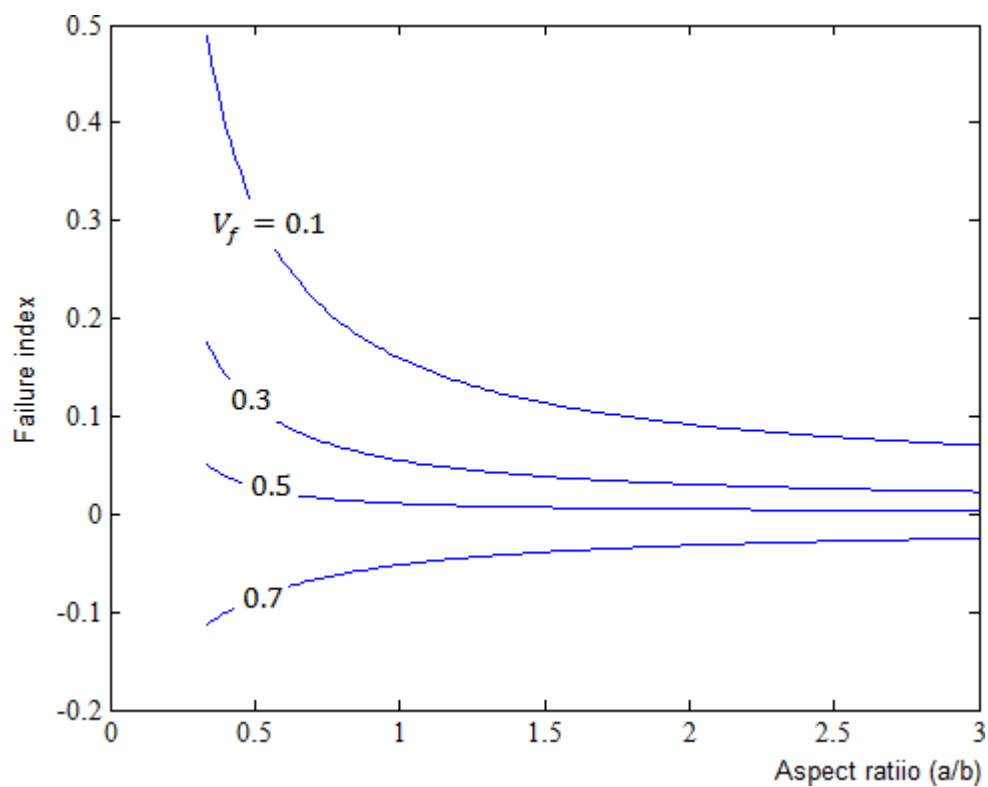


Figure 9. Failure index with aspect ratio at different fiber volume fraction of composite

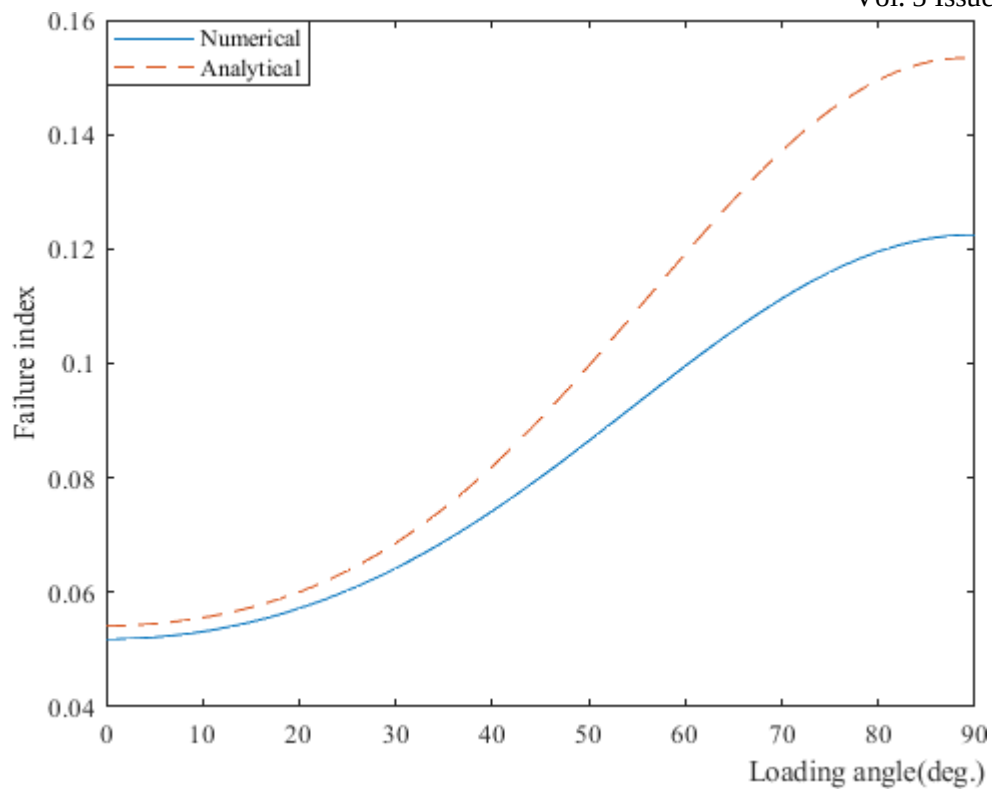


Figure 10. Failure index with loading angles for different technique at ($\nu f f = 0.2, \theta\theta = 0^\circ, b = 1, a = 3$)

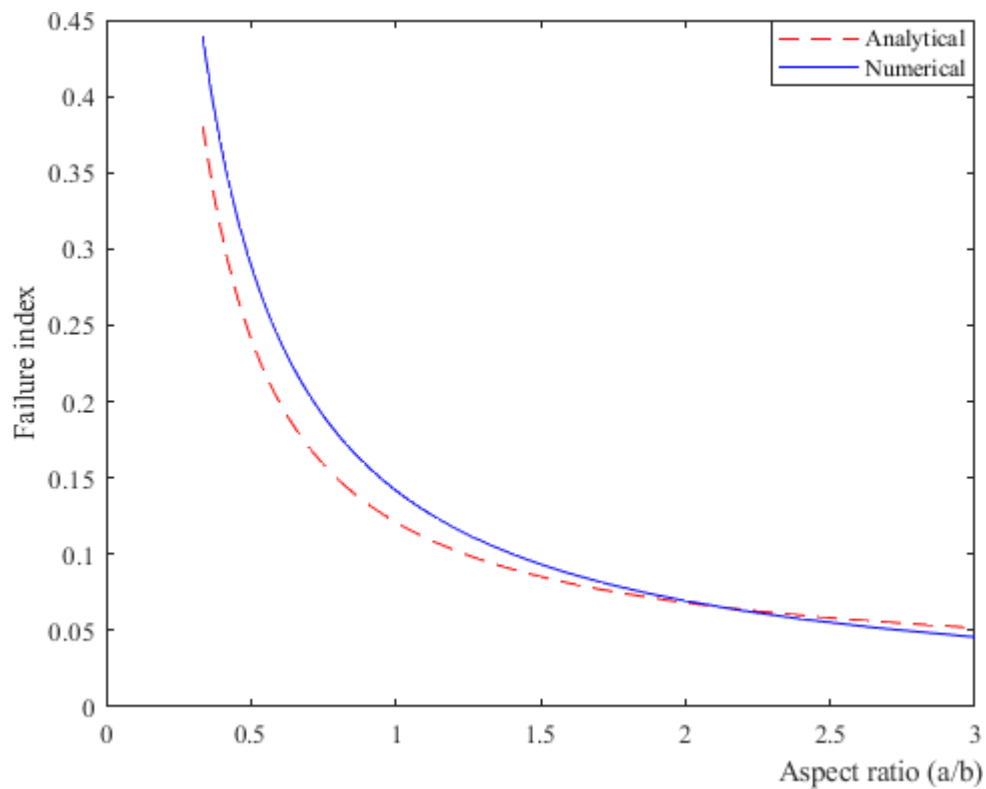


Figure 11. Failure index with aspect ratio for different technique ($\nu f f = 0.2, a l f f = 0, \theta\theta = 0^\circ$)



CONCLUSIONS

In this work, failure behavior of composite plate (E-glass/Epoxy) with a central hole is investigated analytically

and numerically under the effects of various factors. From the above mentioned discussions it is important to note that these;

The maximum failure index of the composite plate can be significantly varied by using appropriate fiber volume fraction and dimension of the hole with respect to the width of the plate.

The utmost value of failure index utilizes at a ply angle of $\theta = 90^\circ$ for E-glass/Epoxy plate, as the fiber orientation decreases, the stress ratio also decreases until to reach a constant value.

As the fiber volume fraction increases, the failure index decreases. However, the rate of these decreases is not constant.

In all cases, the utmost failure index occurs at the corners of the elliptical hole. As the aspect ratio of the elliptical hole decreases, the failure index increases.

REFERENCES

- [1] B. Pal, J. Singh, "Finite Element Flexural Analysis of Sandwich Plates Under Various Types of Loading Conditions", Birla Institute of Technology, Mesra, Ranchi, 2015.
- [2] C.B. Hwu, W.J. Yen, "Green's Function of Two-Dimensional Anisotropic Plates Containing an Elliptic Hole", National Cheng Kung University, 1990.
- [3] J.H. Lee, M.H. Cho, "Analytical Asymptotic Solutions for Rectangular Laminated Composite Plates", IJASS, Korea, 2011.
- [4] Z. Ullah, A. Ali, R.A. Khan, M. Iqbal. "Existence Results to A Class of Hybrid Fractional Differential Equations". *Matriks Sains Matematik*, vol. 1, no. 1, pp. 13-17, 2018.
- [5] L. Xu. "Study on PC Continuous Girder Mechanical Properties Based on Tendon Tensioning Pattern". *Acta Mechanica Malaysia*, vol. 1, no. 1, pp. 12-15, 2018.
- [6] B.Q. Li, Z. Li. "The Implement of Wireless Responder System Based on Radio Frequency Technology". *Acta Electronica Malaysia*, vol. 2, no. 1, pp. 15-17, 2018.
- [7] M. Elmnifi, M. Alshelmany, M. Alhammaly, O. Imrayed. "Energy Recovery from Municipal Solid Waste Incineration Benghazi - Case Study". *Engineering Heritage Journal*, vol. 2, no. 1, pp. 19-23, 2018.
- [8] S.D. Zadeh, G.I. Lvov, "Stress Analysis in an Infinite Hydroxyapatite /Titanium Plate with a Pressurized Circular Hole", *Vestnik Udmurtskogo Universiteta, Mechanics*, vol. 25, no. 2, 2015.
- [9] K.K. Jin, Y. Huang, Y.H. Lee, S.K. Ha, "Distribution of Micro Stresses and Interfacial Tensions in Unidirectional Composites", *Journal of Composite Materials*, vol. 42, no. 18, 2008.
- [10] Z. Vnučec, "Analysis of the Laminated Composite Plate under Combined Loads", *5th International Scientific Conference on Production Engineering*, pp. 143-148, 2005.
- [11] R. Bailer, R. Hicks, "Behavior of



Perforated Plates Under Plane Stress”, 1982.

- [12] P. Mirji, “Optimization of Rectangular Plate with Two Holes Subjected to In-Plane, 2013.
- [13] J. Rezaeepazhand, M. Jafar, “Stress Analysis of Composite Plates with Square Cutout”, University of Mashhad, Mashhad, Iran, vol. 29, no. 13, 2010.
- [14] M. W. Hyer, “Stress Analysis of Fiber-Reinforced Composite Materials”, WCB/McGraw-Hill, New York, 1998.
- [15] M. Roy, “Failure Analysis of Composite Laminates with A Hole by Using Finite Element Method, University of Texas”, Master of Science in Mechanical Engineering, 2005.
- [16] F.C. Campbell, “Introduction to Composite Materials, Structural Composite Materials, ASM International, 2010.
- [17] P. Mirji, “Optimization of Rectangular Plate with Two Holes Subjected to In-Plane Static Loading”, *International Journal of Science & Engineering Research*, vol. 4, no. 6, 2013.
- [18] R. Manoharan, A.K. Jeevanantham, ARPN, “Stress and Load-Displacement Analysis of Fiber Reinforced Composite Laminates with A Circular Hole Under Composite Load, *Journal of Engineering and Applied Science*, 2011.